
Sheet (2)... Series Resonance

1. A series RLC network has R=2kΩ, L=40 mH, and C=1μF. Calculate the impedance at resonance and at one-fourth, one-half, twice, and four times the resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40\times10^{-3})(1\times10^{-6})}} = 5\ \text{krad/s}$$

$$Z(\omega_0) = R = \underline{\mathbf{2\ k\Omega}}$$

$$Z(\omega_0/4) = R + j\left(\frac{\omega_0}{4}L - \frac{4}{\omega_0 C}\right)$$

$$Z(\omega_0/4) = 2000 + j\left(\frac{5\times10^3}{4}\cdot 40\times10^{-3} - \frac{4}{(5\times10^3)(1\times10^{-6})}\right)$$

$$Z(\omega_0/4) = 2000 + j(50 - 4000/5)$$

$$Z(\omega_0/4) = \underline{\mathbf{2 - j0.75\ k\Omega}}$$

$$Z(\omega_0/2) = R + j\left(\frac{\omega_0}{2}L - \frac{2}{\omega_0 C}\right)$$

$$Z(\omega_0/2) = 2000 + j\left(\frac{(5\times10^3)}{2}(40\times10^{-3}) - \frac{2}{(5\times10^3)(1\times10^{-6})}\right)$$

$$Z(\omega_0/2) = 200 + j(100 - 2000/5)$$

$$Z(\omega_0/2) = \underline{\mathbf{2 - j0.3\ k\Omega}}$$

$$Z(2\omega_0) = R + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right)$$

$$Z(2\omega_0) = 2000 + j\left((2)(5\times10^3)(40\times10^{-3}) - \frac{1}{(2)(5\times10^3)(1\times10^{-6})}\right)$$

$$Z(2\omega_0) = \underline{\mathbf{2 + j0.3\ k\Omega}}$$

$$Z(4\omega_0) = R + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right)$$

$$Z(4\omega_0) = 2000 + j\left((4)(5\times10^3)(40\times10^{-3}) - \frac{1}{(4)(5\times10^3)(1\times10^{-6})}\right)$$

$$Z(4\omega_0) = \underline{\mathbf{2 + j0.75\ k\Omega}}$$

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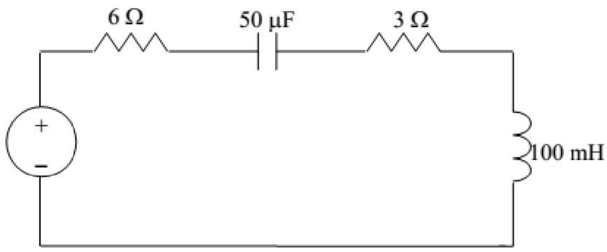
2. A coil with resistance 3Ω and inductance 100 mH is connected in series with a capacitor of 50 pF, a resistor of 6Ωand a signal generator that gives 110 V rms at all frequencies. Calculate ωo, Q, and B at resonance of the resultant series RLC circuit.



$$R = 6 + 3 = 9\ \Omega$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-3}\ x50x10^{-12}}} = \underline{447.21\ \text{krad/s}}$$

$$Q = \frac{\omega_o L}{R} = \frac{447.21x10^3\ x100x10^3}{9} = \underline{4969}$$

$$B = \frac{\omega_o}{Q} = \frac{447.21x10^3}{4969} = \underline{90\ \text{rad/s}}$$

3. Design a series RLC circuit with B=20 rad/s and ω_0=1000 rad/s. Find the circuit's Q.

Let $R = 10\,\Omega$.

$$L = \frac{R}{B} = \frac{10}{20} = 0.5\ \text{H}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(1000)^2(0.5)} = 2\ \mu\text{F}$$

$$Q = \frac{\omega_0}{B} = \frac{1000}{20} = 50$$

Therefore, if $R = 10\,\Omega$ then

$L = \underline{\mathbf{0.5\ H}}$, $C = \underline{\mathbf{2\ \mu F}}$, $Q = \underline{\mathbf{50}}$

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4. Consider the circuit of Figure 1

   a. Determine the values of R and C such that the circuit has a resonant frequency of 25 kHz and an rms current of 25 mA at resonance.

   b. Calculate the power dissipated by the circuit at resonance.

   c. Determine the phasor voltages, V_C, V_L, and V_R at resonance.

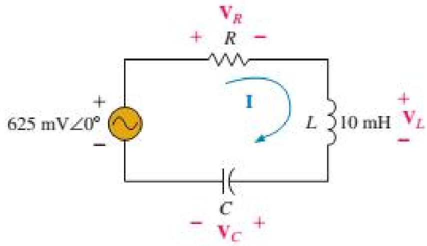
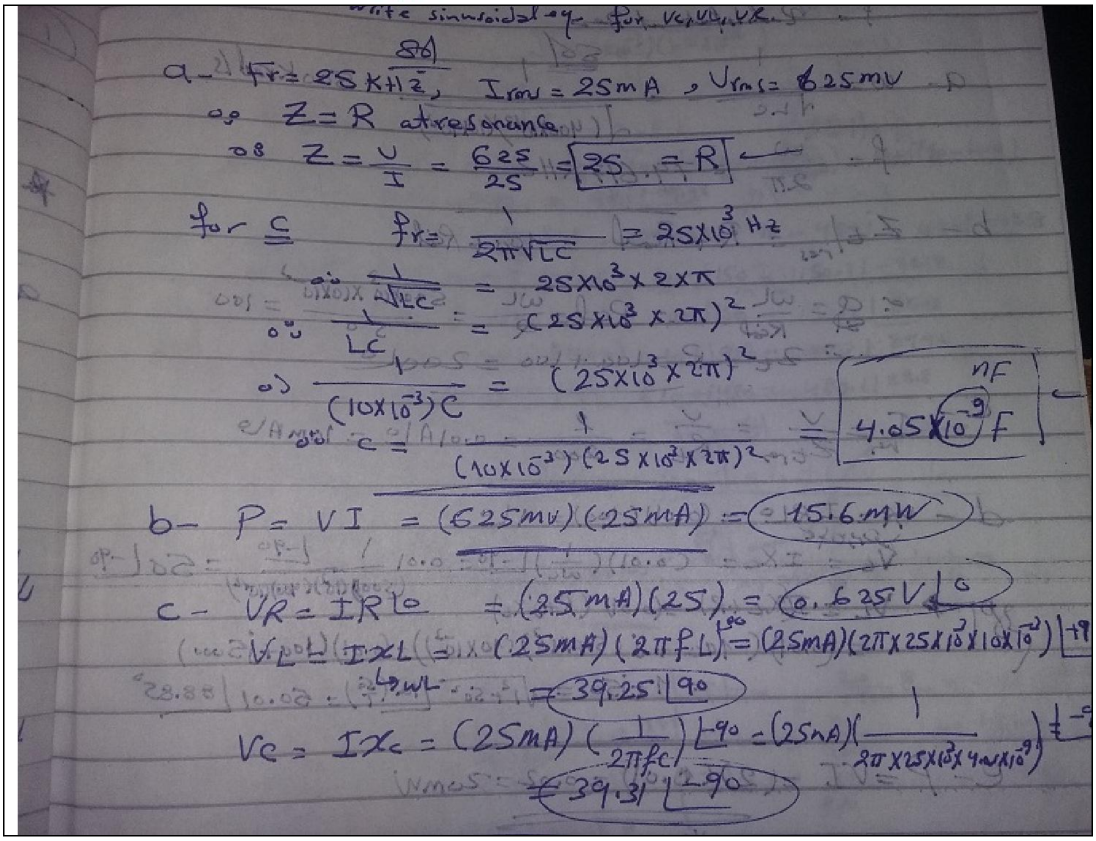


Figure (1)



write sinusoidal eq. for V_C, V_L, V_R

a- $f_r = 25\,kHz$, $I_{rms} = 25\,mA$, $V_{rms} = 625\,mV$

∴ $Z = R$ at resonance

∴ $Z = \frac{V}{I} = \frac{625}{25} = 25 = R$

for C: $f_r = \frac{1}{2\pi\sqrt{LC}} = 25\times10^3\,Hz$

∴ $\frac{1}{\sqrt{LC}} = 25\times10^3 \times 2\times\pi$

∴ $\frac{1}{LC} = (25\times10^3\times2\pi)^2$

∴ $\frac{1}{(10\times10^{-3})C} = (25\times10^3\times2\pi)^2$

$C = \frac{1}{(10\times10^{-3})(25\times10^3\times2\pi)^2} = 4.05\times10^{-9}\,F$ (nF)

b- $P = VI = (625\,mV)(25\,mA) = 15.6\,mW$

c- $V_R = IR\angle0 = (25\,mA)(25) = 0.625\,V\angle0$

$V_L = IX_L = (25\,mA)(2\pi f L)\angle90 = (25\,mA)(2\pi\times25\times10^3\times10\times10^{-3})\angle90$ ($X_L = \omega L$)

$= 39.25\angle90$

$V_C = IX_C = (25\,mA)\left(\frac{1}{2\pi f C}\right)\angle-90 = (25\,mA)\left(\frac{1}{2\pi\times25\times10^3\times4.05\times10^{-9}}\right)\angle-90$

$= 39.31\angle-90$

5. Refer to the circuit of Figure 2.

   a. Determine the resonant frequency expressed as ω (rad/s) and f(Hz).

   b. Calculate the total impedance, Z_T, at resonance.

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c. Solve for current I at resonance.

d. Solve for V_R, V_L, and V_C at resonance.

e. Calculate the power dissipated by the circuit and evaluate the reactive powers, Q_C and Q_L.

f. Find the quality factor, Q_S, of the circuit.

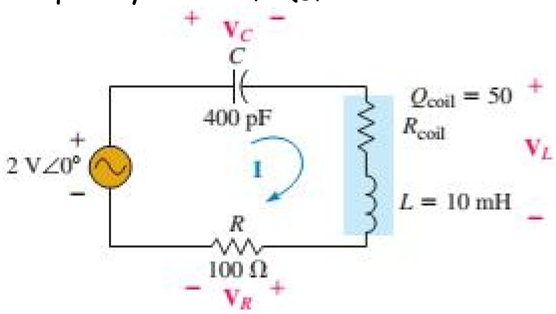
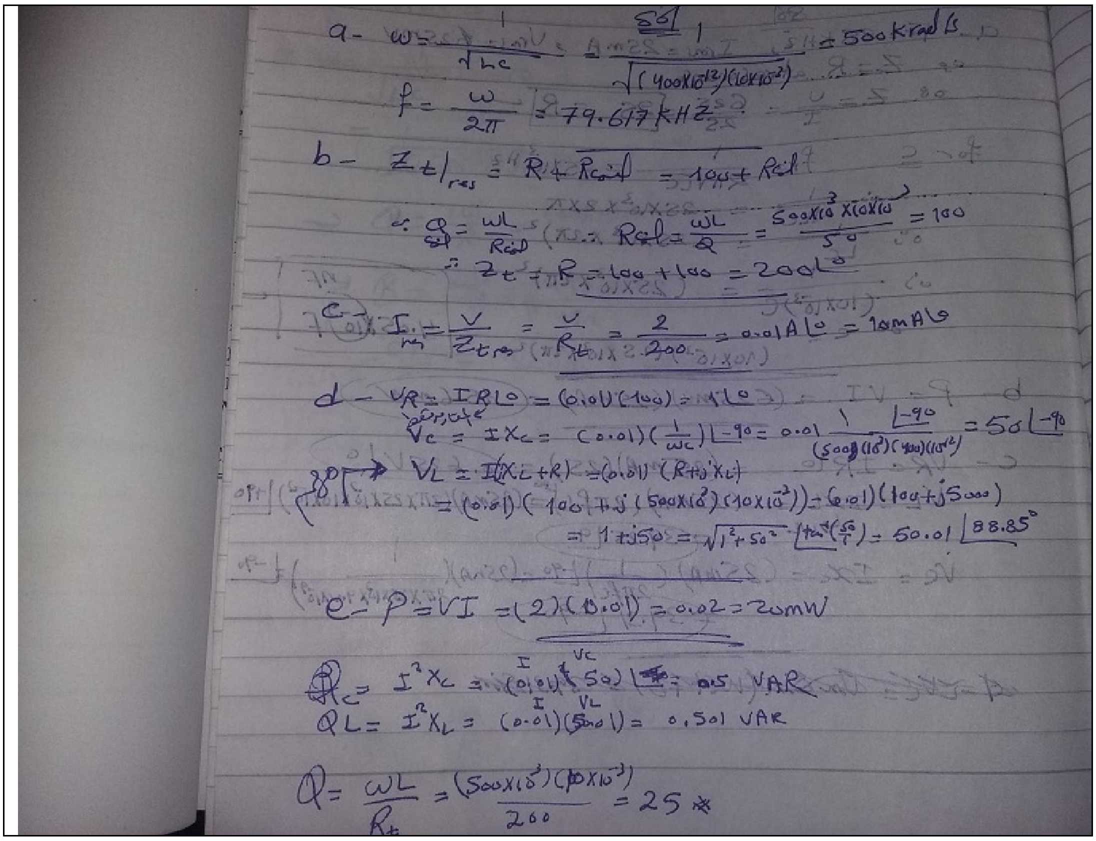


Figure (2)



a- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(400\times10^{-12})(10\times10^{-3})}} = 500\ krad/s$

$f = \frac{\omega}{2\pi} = 79.617\ kHz$

b- $Z_t|_{res} = R + R_{coil} = 100 + R_{coil}$

$\because Q = \frac{\omega L}{R_{coil}}$, $R_{coil} = \frac{\omega L}{Q} = \frac{500\times10^{3}\times10\times10^{-3}}{50} = 100$

$\therefore Z_t = R = 100 + 100 = 200\angle 0$

c- $I_m = \frac{V}{Z_{t\,res}} = \frac{V}{R_t} = \frac{2}{200} = 0.01\,A\angle 0 = 10\,mA\angle 0$

d- $V_R = IR\angle 0 = (0.01)(100) = 1\angle 0$

$V_C = IX_C = (0.01)(\frac{1}{\omega C})\angle -90 = 0.01 \frac{1}{(500)(10^3)(400)(10^{-12})}\angle -90 = 50\angle -90$

$V_L = I(X_L + R) = (0.01)(R + jX_L)$

$= (0.01)(100 + j(500\times10^{3})(10\times10^{-3})) = (0.01)(100 + j5000)$

$= 1 + j50 = \sqrt{1^2 + 50^2}\angle \tan^{-1}(\frac{50}{1}) = 50.01\angle 88.85°$

e- $P = VI = (2)(0.01) = 0.02 = 20\,mW$

$Q_C = I^2X_C = (0.01)(50) = 0.5\ VAR$

$Q_L = I^2X_L = (0.01)(50.01) = 0.501\ VAR$

$Q = \frac{\omega L}{R_t} = \frac{(500\times10^{3})(10\times10^{-3})}{200} = 25$

6. Refer to the circuit of Figure 3.

   a. Find ω_S, Q, and BW (in radians per second).

   b. Calculate the maximum power dissipated by the circuit.

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c. From the results obtained in (a) solve for the approximate half-power frequencies, ω1 and ω2.

d. Calculate the actual half-power frequencies, ω1 and ω2, using the component values and the appropriate equations.

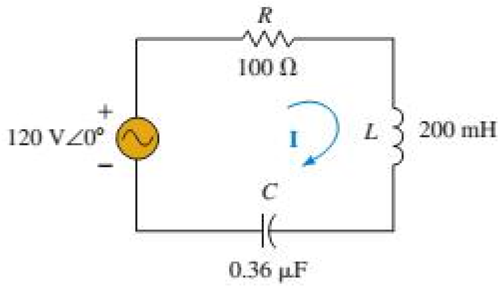
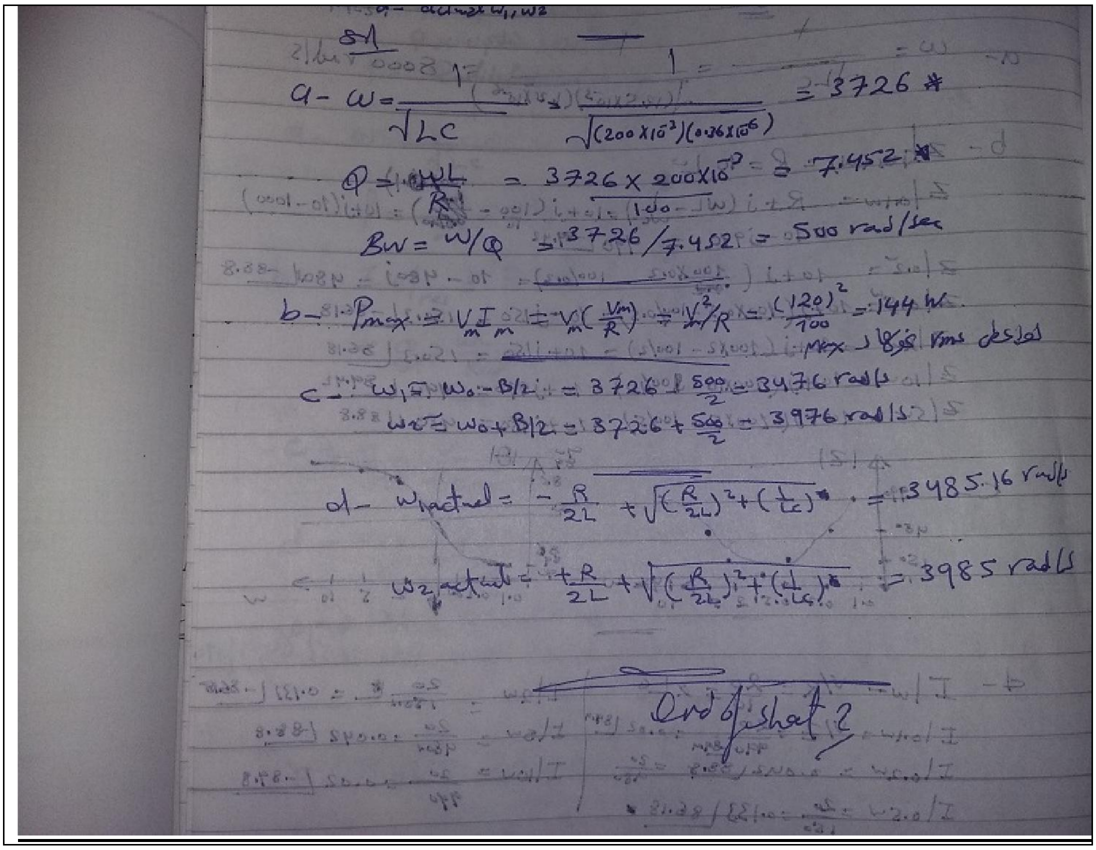


Figure (3)



Sol

a- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(200\times10^{-3})(0.36\times10^{-6})}} = 3726$

$Q = \frac{\omega L}{R} = \frac{3726\times200\times10^{-3}}{100} = 7.452$

$BW = \omega/Q = 3726/7.452 = 500$ rad/sec

b- $P_{max} = V_m I_m = V_m\left(\frac{V_m}{R}\right) = V_m^2/R = \frac{(120)^2}{100} = 144$ W

c- $\omega_1 = \omega_0 - B/2 = 3726 - \frac{500}{2} = 3476$ rad/s

$\omega_2 = \omega_0 + B/2 = 3726 + \frac{500}{2} = 3976$ rad/s

d- $\omega_{1\,actual} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} = 3485.16$ rad/s

$\omega_{2\,actual} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} = 3985$ rad/s

End of sheet 2

Home Assignment (1):

1. For the circuit in Fig. 4, find the frequency ω for which v (t) and i (t) are in phase.

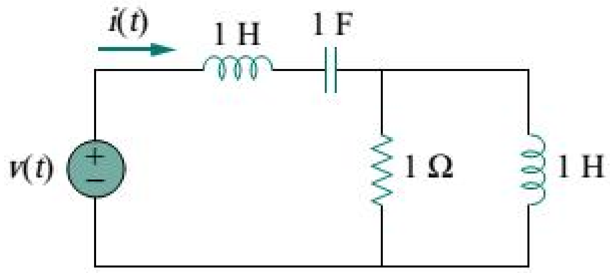


Figure (4)

2. Refer to the series resonant circuit of Figure 5.
   a. Determine the resonant frequency, ω_S.
   b. Solve for the input impedance, $Z_T = Z\angle\theta$, of the circuit at frequencies of 0.1 ω_S, 0.2 ω_S, 0.5 ω_S, ω_S, 2 ω_S, 5 ω_S, and 10 ω_S.
   c. Using the results from (b), sketch a graph of Z (magnitude in ohms) versus ω (in radians per second) and a graph of θ (in degrees) versus ω (in radians per second).
   d. Using your results from (b), determine the magnitude of current at each of the given frequencies.
   e. Use the results from (d) to plot a graph of I (magnitude in amps) versus ω (in radians per second).

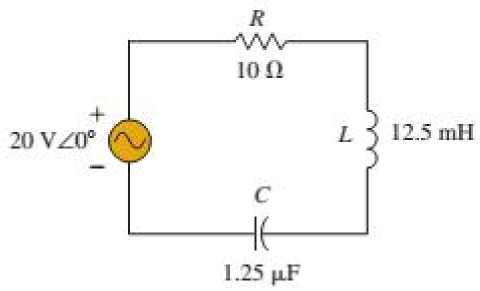
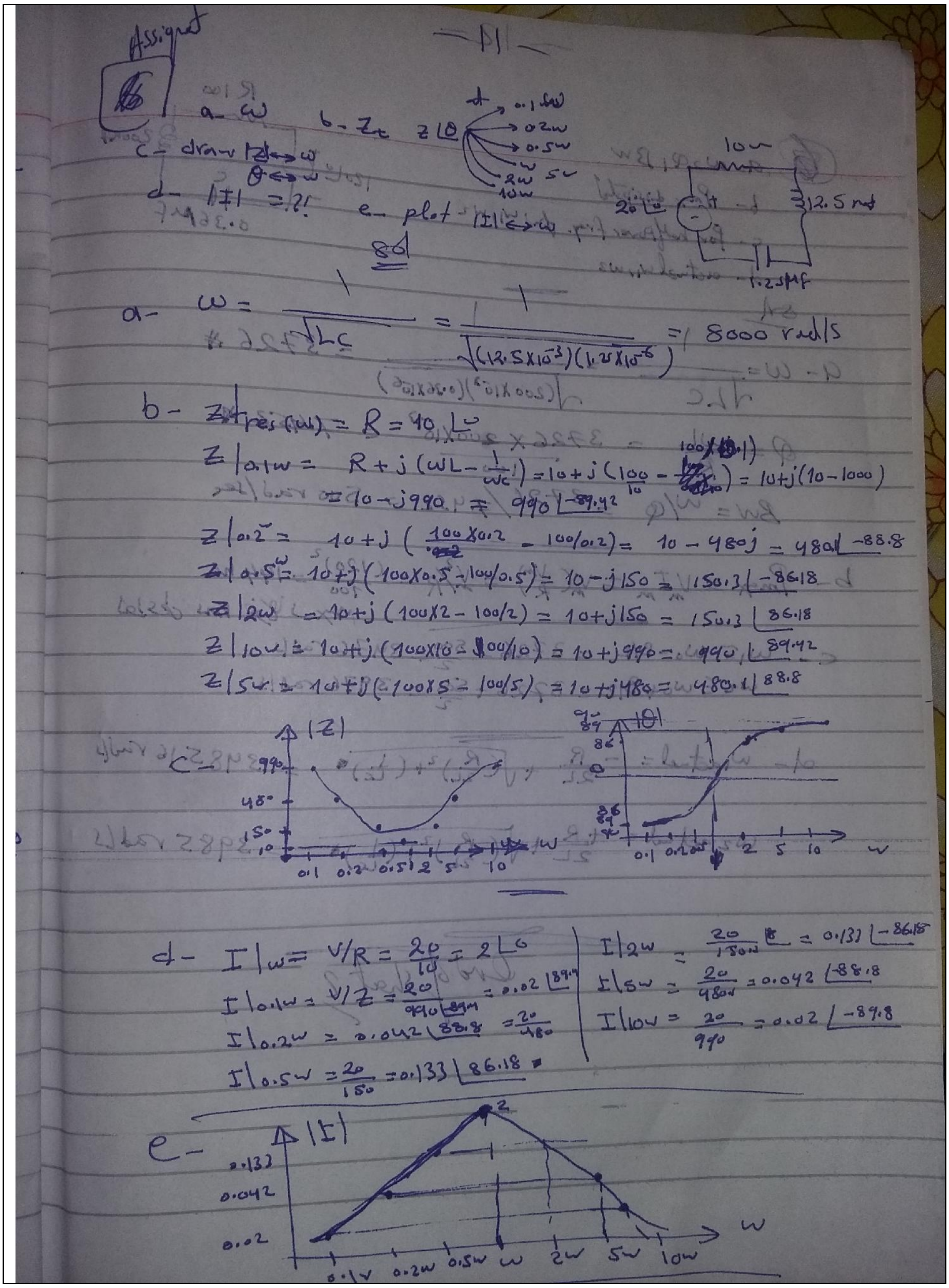


Figure (5)



Assign 6

a- ω b- Z_t Z∠θ → 0.1ω, 0.2ω, 0.5ω, ω, 2ω, 5ω, 10ω

c- draw |Z| ↔ ω, θ ↔ ω

d- |I| = ?! e- plot |I| ↔ ω

10Ω, 20∠0, 12.5 mH, 1.25 μF

sol

a- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(12.5\times10^{-3})(1.25\times10^{-6})}} = 8000$ rad/s

b- $Z_{res}(\omega) = R = 10\angle 0$

$Z|_{0.1\omega} = R + j(\omega L - \frac{1}{\omega C}) = 10 + j(\frac{100}{10} - \frac{100}{0.1}) = 10 + j(10 - 1000)$

$= 10 - j990 = 990\angle -89.42$

$Z|_{0.2\omega} = 10 + j(\frac{100\times0.2}{} - 100/0.2) = 10 - 480j = 480\angle -88.8$

$Z|_{0.5\omega} = 10 + j(100\times0.5 - 100/0.5) = 10 - j150 = 150.3\angle -86.18$

$Z|_{2\omega} = 10 + j(100\times2 - 100/2) = 10 + j150 = 150.3\angle 86.18$

$Z|_{10\omega} = 10 + j(100\times10 - 100/10) = 10 + j990 = 990\angle 89.42$

$Z|_{5\omega} = 10 + j(100\times5 - 100/5) = 10 + j480 = 480.1\angle 88.8$

c- |Z|: 990, 480, 150, 10 vs 0.1, 0.2, 0.5, ω, 2, 5, 10 ; θ: 89, 86, 0, -86, -89 vs 0.1, 0.2, 0.5, ω, 2, 5, 10

d- $I|_{\omega} = V/R = \frac{20}{10} = 2\angle 0$

$I|_{0.1\omega} = V/Z = \frac{20}{990\angle -89.4} = 0.02\angle 89.4$

$I|_{0.2\omega} = 0.042\angle 88.8 = \frac{20}{480}$

$I|_{0.5\omega} = \frac{20}{150} = 0.133\angle 86.18$

$I|_{2\omega} = \frac{20}{150} = 0.133\angle -86.18$

$I|_{5\omega} = \frac{20}{480} = 0.042\angle -88.8$

$I|_{10\omega} = \frac{20}{990} = 0.02\angle -89.8$

e- |I|: 2, 0.133, 0.042, 0.02 vs 0.1ω, 0.2ω, 0.5ω, ω, 2ω, 5ω, 10ω

Good Luck

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